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ORIGINAL ARTICLE

The product-mix problem for multiple production lines in sequenced stages: a case study in the steel industry

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Abstract The product-mix problem is common and widely applicable in many industries. This paper formulates the product-mix problem for multiple production lines in sequenced stages where the output of the upstream stage is the input for other downstream operations. The objective is to maximize the upstream production (throughput per time) by which throughput of the whole system including the downstream operations will be maximized to satisfy production constraints: productivity, capacity, and sales requirements. The problem is formulated using an integer linear programming (ILP) model and demonstrated in a steel plant in Saudi Arabia. The 480-MT steel plant has six production lines, namely push-pickling line, cold-reversing mill, batch-annealing facility, temper mill, continuous galvanizing line, and color-coating line. The plant intended to leverage its annual production capacity. Hence, a recent expansion was done on the batch annealing facility. After the expansion, the plant had several production interruptions and stoppages in the downstream facilities. The model provided the maximum throughput for all production stages and avoids production interruptions.

Keywords Product mix · Linear programming · Practice of OR · Steel industry

1 Introduction

Nowadays, manufacturers strive to enhance flexibility in their production plans in order to face frequent market changes. This requires a production plan to be dynamic and updated continuously. Many times, other situations may also require changes in the production plan, e.g., the emergence of a new technology or the automation of a new production line. The production plan is known to be a hierarchal process where the yearly product demand by the market is cascaded down into monthly and daily production schedules at the shop-floor level. The products are usually grouped into families sharing the same specifications to be manufactured during a period of time. These product groups are addressed in the literature as the product mix, which can be defined in quantities or in percentages of total production.

Since 1960s, the product-mix problem is considered in many applications with industrial contexts. Jaedicke [1] developed one of the earliest models of the product-mix problem based on breakeven analysis (maximizing total profit in the case of multi-product companies). The optimal product mix should satisfy constraints on production or sales. In addition, financial considerations also have been considered (cash flows with interest expenses, debt covenants, and/or the time value of money) for product-mix linear programming models as the work of Albright et al. [2] and Hilton et al. [3]. In recent years, the product-mix problem has been developed with new business concepts. Tsai and Hung [4] proposed a fuzzy goal programming (FGP) approach that integrates activity-based costing (ABC) and performance evaluation in a value chain structure green supply chain (GSC) supplier selection and flow allocation. The FGP model includes flexible goals, financial and non-financial measures, quantitative and qualitative methods, multi-layer structure, multiple criteria, multiple objectives, and multiple strategies. A GSC of a mobile phone is

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used as an illustrative case with several objective structures. In general, the mathematical modelling approach provides the flexibility to include environmental constraints into the product mix problem. The problem can be referred as green product mix problem Tsai et al. [5] and Tsai et al. [6]. Tsai and Hung [7] focused on the treatment and recycling system of waste electrical and electronic equipment (WEEE) reverse logistics management. A two-stage multi-objective decision framework is proposed. In the treatment stage, the responsible producer selects treatment suppliers under extended producer responsibility by a preemptive goal programming model. After the wastes are depolluted and transformed to recyclable materials, the process enters the recycling stage in which the decision-maker plans the reclaimed material production by a linear programming model for profit maximization. The decision model with environmental constraints can reduce the production of the products that have more environmental impacts. Nazari-Shirkouhi et al. [8] integrated the product-mix decisions with the option for outsourcing. It is assumed that the enterprise meets the market demand by producing products inside enterprise and by outsourcing. Fernandes et al. [9] provided a model, using theories from finance, production management, and product offering management, to conclude that there is a relevant difference between the evaluation of the technology that is to be chosen and the potential value due to product-mix adaptations that are able to provide the maximum return from investment. The product-mix problem is suggested to help management make investment decisions related to the flexibility of the manufacturing process to increase customer satisfaction.

In practice, the product mix is defined based on the sales requirements and the production capacity (taking into consideration unavailability due to breakdowns and maintenance requirements). The problem can be formulated as a mathematical model that indeed will provide optimal solutions in many cases. Optimizing product mix determines how productive resources are allocated among various operations to avoid stoppage and interruption of production lines. A manufacturer should identify the economical product quantities that maximize profitability, productivity, and resources. In general, the problem is modeled to maximize throughput, satisfying machine availability, and market demand constraints. However, an alternate approach, for the product-mix problem, is to consider multiple objectives by applying multi-criteria optimization strategies and methods used widely in engineering design. The objective function may cover simultaneously different operational measures including demand fulfillment or resource utilization. Therefore, aggregate measures for different products are generated to cover all objectives and to prioritize products. These models are widely common in semiconductor industry [10–13]. In many cases, the product mix is often defined subjectively or analytically. The methods commonly used for this definition are integer linear programming (ILP) or heuristics based in theory of constraints (TOC), which use maximum throughput as a performance measure [14]. The solution methodology may also include metaheuristic approaches such as genetics algorithms, tabu search, etc. Hasuike and Ishii [15] consider non-linear product-mix problems including randomness of future returns, ambiguity of coefficients, and flexibility of upper value with respect to each constraint such as budget, human resource, time, and several costs. The non-linear programming problems were transformed into deterministic equivalent problems. The product-mix problem also can be set with fuzzy theory and solved using metaheuristic techniques, e.g., simulated annealing (SA) [16] and [17].

The application of the problem is wide and faced in many industries. The steel industry is no exception to this. The steel industry is an intensive capital investment driven by increasing international consumption. The production has increased to cope with the increasing demand, reaching a global annual production of 800 million metric tons. The use of operation research techniques in the steel industry is also observed in the literature [18–20]. Steel products are treated in different production lines in sequence, where the downstream for one production line is the upstream for another. The product mix is defined as a percentage of the annual production capacity of the plant. The products are grouped into families that share the same thickness.

In this paper, we address a steel production plant that had a recent capacity expansion in one production line (the plant has many production lines). Having an expansion in one production line disturbed the product-mix plan, causing the stoppage of production in the upstream lines, although their annual capacity of the upstream should accommodate the expansion capacity. The problem is formulated and solved using an integer linear programming (ILP) model. The model defined the optimal product mix by satisfying the plant's productive, economic, and strategic constraints.

The paper is structured as follows: Sect. 2 reviews the literature, Sect. 3 provides model notations, Sect. 4 discusses the model formulation, Sect. 5 gives the model solution methodology, Sect. 6 details the numerical analysis of the case under the study, Sect. 7 presents the results, and finally, Sect. 8 presents the concluding remarks.

2 Literature review

Several algorithms to determine the product mix under the TOC have been developed. Most of the previous works focused on one bottleneck (dominant bottleneck) and considered the product-mix problem with exact data. Badri et al. [10] used all bottlenecks in order to determine the aggregated priority of each product, to propose a multi-criteria decision-making approach for product-mix problem with interval parameters. The proposed approach involves application of interval technique for order of preference by similarity to ideal solution (TOPSIS) to calculate the aggregated priority of each product and use them to improve the product-mix plan. Chung et al. [11] used analytic hierarchy process (AHP) and analytic network process (ANP) approaches to analyze multiple process inputs and outputs, incorporating experts' opinion on their priority of importance, to obtain optimal product mixes for semiconductor production. Lee et al. [12] constructed an analytical approach for dealing with the managerial problems under subjective judgment environments in the semiconductor industry. A fuzzy AHP method is utilized to deal with uncertainty of the criteria of product, equipment efficiency. Then, a priority mix of strategic alternatives represented by a priority index can be evaluated. In recent years, product-mix problem gained the attention of many researchers due to its wide applications in many industries. Romauch and Klemmt [13] addressed the product-mix problem in semiconductor fab production to meet different objectives (demand and operational constraints). Tai et al. [21] addressed the multi-product-mix problem in thin-film transistor liquid crystal display manufacturing, which included a multi-stage production process under three main constraints—product mix, available capacitates, and resource utilization—to minimize the cost of production.

Wu et al. [22] developed a non-linear mathematical program to model and solve the product-mix decision problem for a mixed-yield scenario in semiconductor manufacturing, which involves the simultaneous production of high-yield and low-yield products. Two methods for solving the non-linear program are proposed. Method 1 converts the non-linear program into a linear program by setting some variables as parameters. Method 2 aims to reduce the computation complexity while providing a near optimal solution. Experiment results show that method 2 is better than method 1, when aggregately considering solution quality and computation efforts. Tanhaei and Nahavandi [23] presented an algorithm to determine the product mix in a two-constraint resources environment. The TOC solution could not reach optimum solution and has the risk of being infeasible when multiple constraint resources exist. Tanhaei and Nahavandi [23] algorithm is suitable for improving solutions obtained from TOC and could provide throughput in product-mix problems. Some alternatives are compared: the standard TOC, ILP, tabu search, hybrid tabu SA, and proposed algorithm solution which showed a better results.

A trend has also been observed in the literature to emphasize the role of activity-based costing (ABC) in formulating the product-mix problem. Malik and Sullivan [24] developed a mixed-integer programming model that utilizes ABC information to determine optimal product mix and product cost in a multi-product manufacturing environment. The model permits interesting insights to be gained into the evaluation of marginal cost of products and marginal worth of resources for decision making involving product mix, product costing, and capacity expansion/contraction. Kee and Schmidt [25] model the selection of a product mix with the theory of constraints (TOC) and an ABC model with the capacity of production-related activities. The work demonstrates that management's discretionary power over labor and overhead resources determines when the TOC and ABC lead to optimal product-mix decision. Tsai et al. [26] provided a model to help managers to make a product-mix decision and identify excess resources so that managers can optimize resource usage. The work considers the impact of price changes on product-mix decisions and also examines the impact of reducing product price with different price elasticity of demand on the simulated company's profit. Zhuang and Chang [27] proposed a mixed-integer programming (MIP) model, based on the time-driven activity-based costing (TDABC) accounting system. By using a time driver from the resource to the cost objects and simultaneously dealing with numerous resource limitations, the model obtains a global optimal decision. The implications for the use of this accounting system adoption to determine product-mix are shown.

3 Notations

PPL	Push pickling line production capacity
CRM	Cold reversing mill production capacity
CGL	Continuous galvanizing line production capacity
TPM	Temper mill production capacity
BAF	Batch annealing facility production capacity
CCL	Color coating line production capacity
MT	Metric ton
PPL_{x_i}	PPL productivity in metric ton per hour for x_i, $(i = 1, 2, 3, 4, 5, 6)$
CRM_{x_i}	CRM productivity in metric ton per hour for x_i, $(i = 1, 2, 3, 4, 5, 6)$
CGL_{x_i}	CRM productivity in metric ton per hour for x_i, $(i = 1, 2, 3, 4, 5, 6)$
BAF_{x_1}	BAF productivity in metric ton per hour for x_i, $(i = 1, 2, 3, 4, 5, 6)$

4 Mathematical model

Steel plants are composed of many production units where the downstream of one unit is the upstream for another unit. Hence, capacity expansion of one unit will affect other interrelated production units and cause imbalance in the product mix. In general, the "product-mix" model in sequenced stages is formulated in various mathematical programming approaches to determine the decision related to optimal product mix for each stage that maximizes the total profit (throughput) under some limits (sequence of production stages) or constraints on production (sales requirements). The addressed model in this paper will be formulated as follows.

4.1 Assumptions

Without loss of generality, the sequenced stage model is developed assuming six steel products, x_i $(i = 1, \cdots, 6)$, that must pass by several production stages. The sequenced stage product-mix decision model presented includes the following assumptions:

- Each production stage has a different availability during the year (maintenance requirements affect machine availability).
- Each production stage has different productivity rate; hence, production speeds will differ.
- The output of one production stage will the input feed in the next production stage.
- Each product has a sale forecast to satisfy the market demand.
- There is a limited storage area between the production stages. The production stage will stop if the storage area is full as there will be no space to store the production.
- The production stage machine resources cannot be expanded by renting or purchasing additional machines.

4.2 Objective function

In many process industries, the production stages are sequenced in stages. The output of the upstream stage is the input for the downstream operations as shown in Fig. 1. In most of these cases, the productivity (throughput per time) of the sequenced stages is not equal. Hence, capacity expansion of one unit will affect other interrelated production units and cause imbalance in the product mix. The objective of the model is to maximize the production throughput (upstream production). Maximizing the throughput of production is widely common in many product-mix problems (Sobreiro et al. [14]). In the considered system (similar to Fig. 1 layout), the objective function can be considered to maximize the production of the upstream stage (input stage) as it will be the highest throughput in the whole system. Although this system configuration fits a wide range of process industries, it may not be applicable for other industrial contexts. Therefore, the model can be tailored to accommodate the particular needs of other production systems when needed.

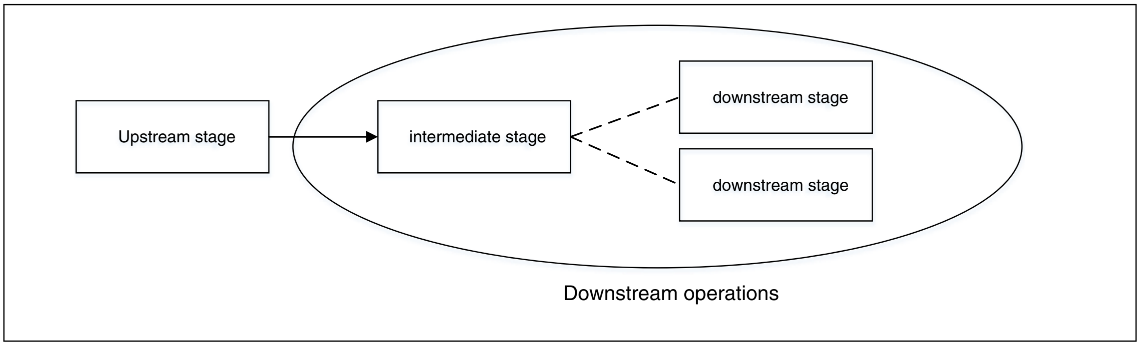

Fig. 1 Production stages

4.3 Constraints

1. Constraints set 1: Productivity constraint for each production stage is formulated as follows:

$$\sum_{i=1}^{6} \frac{x_{i\_Production\ \ stage}}{Productivity_{x_i}} \leq \text{Available Production Hours}$$

where

$x_{i\_Production\ \ stage}$: quantity to be produced in production stage devoted for x_i product in metric ton (MT)

$Productivity_{x_i}$: productivity of production stage devoted for x_i in ton per hour (TPH)

2. Constraint set 2: Each production stage has a certain production capacity. However, to maintain acceptable levels of utilization, each production line should maintain a minimum annual production capacity. The total amount of production for the six products should not exceed the planned production capacity determined for every production line.

$$\sum_{i=1}^{6} x_{i\_Production\ \ Stage} \geq \text{minimum annual production capacity}$$

3. Constraint set 3: Based on the sales forecast, each production stage will be tentatively assigned a production share of each product that covers all six products. To provide some flexibility in the model, the intended product-mix percentage was allowed to be exceeded by no more than ∂ (chosen to be 20 %). This will be determined based on the optimization model.

$$(1-\partial)*\%x_{i\_stage} \leq \left(\frac{x_{i\_stage}}{\sum_{i=1}^{6} x_{i\_stage}} \right) \leq (1+\partial)*\%x_{i\_stage}; \quad (i = 1, \cdots 6)$$

4. Constraint set 4: The maximum throughput in the upstream production stage will be higher than the following production stages in the sequence for the remaining downstream production stages as follows:

> Production in upstream production stage ≥ downstream production stages

5 Solution procedure

The product-mix problem can be solved using different approaches, depending upon several factors, one of which is the type of objective function, and their constraints. These can vary in their mathematical relationship; they can be linear or non-linear. In the literature, it has been proven that integer linear programming (ILP) is considered an effective technique for optimizing the process throughput (Sobreiro et al. [14]; Romauch and Klemmt [13]). Moreover, according to Miltenburg [28] and Aryanezhad and Komijan [29], the ILP should be used to solve the problem of defining product mix when the following applies: the decision variables are the produced units of product, the objective function is the throughput maximization, and the model constraints are the resource capacity available. This work followed the general steps used in mathematical modeling as follows:

1. Data gathering: collect input information of the plant by meeting the production team and sales.
2. Verification of the model: the input data will be verified by observations and reviewing previous production reports.
3. Model solution: after validation, the model will be coded and solved using the General Algebraic Modeling System (GAMS) software [30]. GAMS was used to input the model, and the Branch-And-Reduce Optimization Navigator (BARON) solver was used to reach the optimal solution. The BARON solver is a computational system designed initially for solving non-convex NLP and integer programming optimization problems to global optimality. BARON implements deterministic global optimization algorithms of the branch-and-bound type that are guaranteed to provide global optima.
4. Validation of the results: the output results will be validated to ensure the applicability of implementation. Step II may be revisited to modify the model, if needed.

6 Numerical analysis

Over the past few years, due to fluctuations in oil prices, many oil-producing countries started to diversify national income by investing in non-oil investments, e.g., in construction, infrastructure, and large-scale real estate projects. This has increased the demand for steel products and motivated steel manufacturers to leverage their production capacities. In this case study, we show a real-life example of product-mix disturbance in a steel plant due to the expansion in one production unit. The plant is located in Saudi Arabia.

Figure 2 shows the plant's layout contained in one complex (cold mill complex); some units are upstream/downstream from the others.

In May 2013, the Saudi steel plant completed a capacity enhancement project in its batch annealing facility (BAF) by adding a total of 28,000 MT per year to its original design capacity, which was 150,000 MT per year, to allow the facility to produce a total of 178,000 MT per year in an effort to increase its market share in cold-rolled products. Initial calculations showed that BAF expansion was permissible. This was demonstrated by the following calculations. The cold reversing mill (CRM) and push pickling line (PPL) should have been able to supply additional feedstock material to BAF after expansion, as CRM needed to produce a total of 428,000 MT (expanded BAF (178,000) + continuous galvanizing line (CGL) capacity (250,000)). Although this exceeded CRM design capacity, i.e., 420,000 MT, the actual market supply was forecasted to be only 410,000 MT (240,000 for CGL and 170,000 for BAF). Similarly, PPL needed to produce [410,000 + 20,000 (material going directly to customers) = 430,000] yearly, which is far lower than its design capacity (i.e., 480,000 MT). Therefore, the plant's management anticipated that there should not be a problem in the feedstock material; hence, they decided to go with this capacity enhancement project in the BAF production line (adding a total of 28,000 MT per year to reach 178,000-MT annual capacity). Table 1 summarizes the plant's divisions.

After the start-up of the BAF expansion, neither PPL nor CRM could smoothly supply the required quantities to fulfill the BAF and CGL production requirements. Many stoppages and production delays took place in BAF, CGL, and CRM. Consequently, several customers' complaints were received regarding late deliveries; additionally, many orders could not be taken due to out-of-stock status to reduce the load in front of the production lines in order to fulfill current orders' promised dates. CRM was suffering from a lack of feedstock material from PPL mainly due to PPL's productivity not keeping up with the new productivity of CRM. Therefore, it was decided to review the current product mix to solve the problem.

The management decided to review the steel product mix to identify bottlenecks. The products were differentiated based on thicknesses. To solve the problem, the production plan needed to be updated. The plan produced six general classes of products based on their sizes (thicknesses); these products are stated in Table 2. Let x_{i_j}, where $(i = 1, \cdots, 6)$, be the

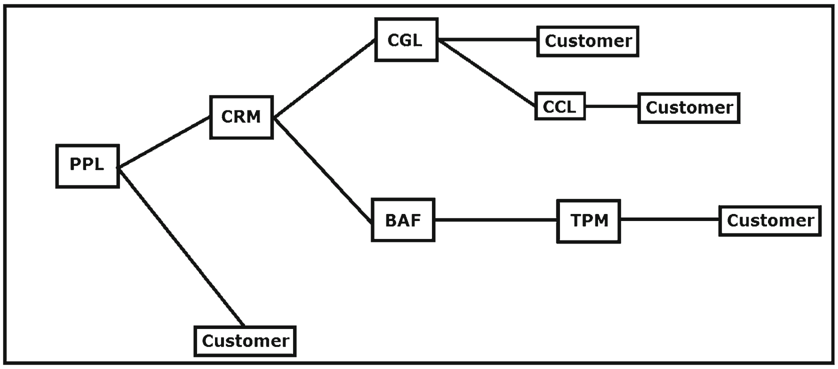

Fig. 2 Plant layout

quantity to be produced in metric ton from x_i product in j production line, where $j = \{PPL, CRM, CGL, BAF\}$.

The objective is to maximize steel throughput with the highest thickness, i.e., thick-gauge material x_6, in the upstream, i.e., produced by the PPL production line.

$$\mathrm{Max}(x_{6\_PPL}) \tag{1}$$

This indeed improved productivity so as to produce fewer thicknesses $(x_5, x_4, x_3, x_2, x_1)$ in the production line. It started production from the thick-gauge material x_6. The model constraints were needed for every product class in order to satisfy the planned product mix. The model revealed product quantities that satisfied production lines' productivities (PPL, CRM, CGL, and BAF). This kept the product mix realistic and not too far from the planned product mix by the sales team. Table 3 illustrates available time and production targets for the production lines based on the 2014 sales and operational plan. Four production lines (PPL, CRM, CGL, and BAF) were decided by the management to be the main focus bottlenecks of the system.

For the four production lines, there were four constraints for the planned production quantity needed to be produced in metric ton; another four constraints were required for the available time in hours for every production line. Table 3 shows the required production amounts and production hours needed for the four production lines.

In addition, the productivity ton per hour (TPH) was also needed for the production lines. Table 4 presents the design productivity for the four production lines in terms of TPH.

Table 1 Plant's production lines

Production line	Production capacity (MT per year)	Description
Push pickling line (PPL)	480,000	PPL's main function is to remove the scale (iron oxide) from steel strip surfaces (top and bottom). This is accomplished by dipping the strip into a hydrochloric acid (HCL) solution; after that, it is rinsed and dried. PPL also involves edge trimming of the strip edges and oiling of both strip surfaces.
Cold reversing mill (CRM)	87.5 % of PPL production, i.e., 420,000	CRM's main function is to reduce strip thickness according to the customers' requirements; such reduction is accomplished in several passes, whereby strip thickness is reduced in several stages. The total reduction percent is approximately 75 %.
Batch annealing facility (BAF)	178,000	BAF's main purpose is to anneal the product in order to achieve the targeted mechanical properties according to customers' requirements; it also involves cleaning the strip surface of dirt and the black rolling oil used in CRM.
Temper mill (TPM)	Same as BAF (178,000)	TPM tempers soft BAF products before they are sent to customers. This process is intended to remove yield-point elongation and prevent stretch strain, impart the desired surface finish to the product, and improve strip flatness.
Continuous galvanizing line (CGL)	250,000	Though CGL's main function is to apply a zinc coat onto both strip surfaces, it involves many other processes: cleaning, annealing, applying chromium film, skin passing, and oiling. CGL products can be supplied directly to customers or can be sent to CCL for further processing (to apply a paint coat to both strip surfaces).
Color coating line (CCL).	90 % of CGL production	This is a precoating line that applies coats through advanced application and baking of coating materials onto strips produced by CGL to enhance corrosion resistance. It is an add-on for visual aspects of strips, such as color and gloss, as well as for protective elements, such as weather and corrosion resistance.

Table 2 Steel product specifications (CRM converts products from PPL's thicknesses to CGL and BAF thicknesses)

Product	Thickness class (mm)	
	Thicknesses at PPL	Thicknesses at CGL and BAF
x_{1_j}	≤1.59	≤0.34
x_{2_j}	1.60–2.00	0.35—0.50
x_{3_j}	2.01–2.49	0.51–0.70
x_{4_j}	2.50–2.99	0.71–0.90
x_{5_j}	3.00–4.49	0.91–1.39
x_{6_j}	≥4.5	≥1.40

There were constraints for the available time in hours for each production line (PPL, CRM, CGL, and BAF).

$$\sum_{i=1}^{6}\frac{x_{i\_PPL}}{PPL_{x_i}}\le 6,109* \tag{2}$$

where

x_{i_PPL}: quantity to be produced in PPL from x_i product in metric ton

PPL_{x_1}: PPL productivity for x_i in TPH (Table 4)

*Note: PPL total available time was 6832 h, but only 6109 h was used in the above constraint because we excluded 723 h for the production of other products going directly to customers.

$$\sum_{i=1}^{6}\frac{x_{i\_CRM}}{CRM_{x_i}}\le 6986 \tag{3}$$

where

x_{i_CRM}: quantity to be produced in CRM from x_i product in metric ton

$CRM_{\_x_i}$: CRM productivity for x_i in TPH (Table 4)

$$\sum_{i=1}^{6}\frac{x_{i\_CGL}}{CGL_{x_i}}\le 7,326 \tag{4}$$

where

Table 3 Available time and production plan for the production lines

Production line	Production amount (MT)	Available time (h per year)
PPL	432,476	6832
CRM	408,290	6986
CGL	237,361	7326
BAF	169,970	7735

Table 4 Design productivity for the production lines

Product class	PPL productivity (TPH)	CRM productivity (TPH)	CGL productivity (TPH)	BAF productivity (TPH)
X1	54	30	19.6	22
X2	54	38	24.4	22
X3	58	52	32.6	22
X4	75	62	35.4	22
X5	75	69	35.9	22
X6	75	79	37.5	22

x_{i_CGL}: quantity to be produced in CGL from x_i product in metric ton

$CGL_{\_x_i}$: CGL productivity for x_i in TPH (Table 4)

$$\sum_{i=1}^{6}\frac{x_{i\_BAF}}{BAF_{x_i}}\le 7,735 \tag{5}$$

where

x_{i_BAF}: quantity to be produced in BAF from x_i product in metric ton

$BAF_{\_x_i}$: BAF productivity for x_i in TPH (Table 4)

The planned production quantity needing to be produced for every production line was formulated as follows:

$$\sum_{i=1}^{6}x_{i\_PPL}\ge 409,510* \tag{6}$$

*Note: PPL total planned production was 432,476 MT, but only 409,510 MT was used in the above constraint because we excluded 22,966 MT for the production of the HRCP (products going directly to customers).

$$\sum_{i=1}^{6}x_{i\_CRM}\ge 408,290 \tag{7}$$

Table 5 Planned product-mix percentages for the production lines

Product class	PPL (%)	CRM (%)	CGL (%)	BAF (%)
X1	1.0	1.0	2.3	0.0
X2	12.3	12.3	14.3	10.0
X3	21.1	21.1	27.4	20.0
X4	19.9	19.9	14.0	24.0
X5	25.2	25.2	21.0	21.0
X6	20.4	20.4	21.0	25.0
Summation	100	100	100	100

Table 6 Results at PPL and CRM upstream

Product class	Thickness (mm)	Production (MT)	Resulted mix (%)	Initial forecast (%)	Difference (%)
x_{1_PPL}	≤1.59	4260	1.0	1.0	0
x_{2_PPL}	1.60–2.00	50,214	12.3	12.3	0
x_{3_PPL}	2.01–2.49	93,169	22.7	21.1	+1.6
x_{4_PPL}	2.50–2.99	81,491	19.9	19.9	0
x_{5_PPL}	3.00–4.49	83,101	20.3	25.2	−4.9
x_{6_PPL}	≥4.5	97,418	23.8	20.4	+3.4
Total		409,653	100.0	100.0	

$$\sum_{i=1}^{6} x_{i\_CGL} \geq 237,361 \tag{8}$$

$$\sum_{i=1}^{6} x_{i\_BAF} \geq 178,168 \tag{9}$$

Table 5 illustrates the planned product mix to be produced from every product based on the 2014 sales and operation plan.

To maintain the product-mix percentages in the upstream production line, i.e., PPL, the following constraints were added. To provide some flexibility in the model, the intended product-mix percentage was allowed to be exceeded by no more than ∂ (chosen to be 20 %), as follows:

$$(1-\partial)*\%x_{i\_PPL} \leq \left(\frac{x_{i\_PPL}}{\sum_{i=1}^{6} x_{i\_PPL}} \right) \leq (1+\partial)*\%x_{i\_PPL} \tag{10}$$

Table 7 Results at CGL

Product class	Thickness (mm)	Production (MT)	Resulted mix (%)	Initial forecast (%)	Difference (%)
x_{1_CGL}	≤0.34	5459.3	2.3	2.3	0
x_{2_CGL}	0.35–0.50	33,942.6	14.30	14.3	0
x_{3_CGL}	0.51–0.70	63,850.1	26.90	27.4	0
x_{4_CGL}	0.71–0.90	35,129.4	14.80	14.0	−0.80
x_{5_CGL}	0.91–1.39	46,048.0	19.40	21.0	1.60
x_{6_CGL}	≥1.40	53,168.8	22.40	21.0	−1.40
Total		237,361	100	100	

Table 8 Results at BAF

Product class	Thickness (mm)	Production (MT)	Resulted mix (%)	Initial forecast (%)	Difference (%)
x_{1_BAF}	≤0.34	890	0.50	0.0	−0.50
x_{2_BAF}	0.35–0.50	16,020	9.00	10.0	1.00
x_{3_BAF}	0.51–0.70	37,914	21.30	20.0	−1.30
x_{4_BAF}	0.71–0.90	41,830	23.50	24.0	0.50
x_{5_BAF}	0.91–1.39	43,076	24.20	21.0	−3.20
x_{6_BAF}	≥1.40	38,804	21.80	25.0	3.20
Total		177,970	100	100	

$$(1-\partial)*\%x_{i\_CRM} \leq \left(\frac{x_{i\_CRM}}{\sum_{i=1}^{6} x_{i\_CRM}} \right) \leq (1+\partial)*\%x_{i\_CRM} \tag{11}$$

$$(1-\partial)*\%x_{i\_CGL} \leq \left(\frac{x_{i\_CGL}}{\sum_{i=1}^{6} x_{i\_CGL}} \right) \leq (1+\partial)*\%x_{i\_CGL} \tag{12}$$

$$(1-\partial)*\%x_{i\_BAF} \leq \left(\frac{x_{i\_BAF}}{\sum_{i=1}^{6} x_{i\_BAF}} \right) \leq (1+\partial)*\%x_{i\_BAF} \tag{13}$$

where $\%x_{i\_PPL}(i=1,\cdots,6)$, $\%x_{i\_CRM}(i=1,\cdots,6)$, $\%x_{i\_CGL}(i=1,\cdots,6)$, $\%x_{i\_BAF}(i=1,\cdots,6)$ are shown in Table 5.

Also, since PPL was the upstream for the following production lines—CRM, CGL, and BAF—then, the following constraints followed:

$$x_{6\_PPL} \geq x_{6\_CRM} \tag{14}$$

$$x_{6\_PPL} \geq x_{6\_CGL} \tag{15}$$

Table 9 Proposed and planned production and available time

Production unit	Proposed (MT)	Actual (MT)	Available time (h)	Required time (h)
PPL	409,561	409,510	6108.6	6108.6
CRM	408,340	408,290	6986.1	6986.1
CGL	237,361	237,361	7326.4	7326.4
BAF	178,168	177,970	7734.9	7734.9

Table 10 PPL actual product mix for 2013

Class	Jan (%)	Feb (%)	March (%)	Apr (%)	May (%)	June (%)	July (%)	Aug (%)	Sep (%)	Oct (%)	Nov (%)	Dec (%)
X1	0.00	3.92	1.76	2.73	2.77	2.55	0.06	0.08	0.19	0.00	0.02	0.00
X2	5.23	10.20	9.90	8.35	7.05	5.23	3.04	5.89	6.09	8.50	4.31	6.04
X3	27.02	17.80	30.22	16.60	25.75	26.64	34.57	27.64	27.13	29.45	29.51	19.70
X4	20.33	20.22	16 .73	21.24	24.38	18.73	27.99	26.52	16.72	25.16	21.62	18.91
X5	24.09	24.09	20.29	24.19	22.16	20.55	17.21	23.62	24.56	22.84	22.75	17.90
X6	23.33	23.77	21.10	26.89	17.89	26.29	17.14	16.26	25.31	14.04	21.79	37.46

$$x_{6\_PPL} \geq x_{6\_BAF} \tag{16}$$

7 Results

Table 6 illustrates the proposed product mix with the maximum production of the x_{6_PPL} class. It also shows the initial product-mix forecast.

Tables 7 and 8 show the results for the CGL and BAF lines.

The proposed production and available time compared to the planned ones for this solution are shown in Table 9.

Table 10 represents the actual product mix for 2013 on a monthly basis. The model solution shows that the maximum production limit for the upstream production line x_{6_PPL} was 97,418 MT (23.8 %), which satisfied all constraints. It shows that x_{6_PPL} violated the maximum allowable limit, which was 23.8 %; thus, stoppages were expected to occur in the downstream production line, CRM. Table 10 also shows PPL production in 2013. Note that the expansion commenced in March 2013. The recommended percentage was violated in 4 months, namely April, June, September, and December. Hence, this contributed to stoppages encountered in 2013 after the expansion.

8 Concluding summary

The main purpose of this paper was to solve the product-mix problem for multiple production lines in sequence. The problem has wide applications in industry and has been found to be useful in the steel industry environment specifically. Steel plants usually have many production lines in sequence. Steel production is motivated to leverage production due to the increasing demands in many parts of the world that include Arabian oil-producing Gulf countries. Hence, many production capacity expansion plans have commenced. The expansion plans may disturb the production where stoppages occur. This paper demonstrated this issue through a case study in Saudi Arabia. After a capacity enhancement project was carried out in a steel production plant, some production lines were not able to supply the additional capacity required, despite having enough capacity as per their design capacities. As a result, many production stoppages were encountered.

This paper focused on the product mix and studied the possibility of how it could affect the performance of all production lines, as well as how such effects could be minimized. There were many reasons for such stoppages; however, the formulation focused only on the product mix and studied the possibility of how it could affect the performance of all production lines, as well as how such effects could be minimized. The model provided the maximum throughput in the upstream production limit that satisfied all constraints. By not exceeding this limit in any month, the stoppages could be avoided. The model was validated by reviewing the production of the previous year. It was found that the recommended percentage was violated in four different months, which caused the stoppages.

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